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ORIGINAL RESEARCH

Linear quadratic control and estimation synthesis for multi-agent systems with application to formation flight

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Abstract

This paper concerns the optimality problem of distributed linear quadratic control in a linear stochastic multi-agent system (MAS). The main challenge stems from MAS network topology that limits access to information from non-neighbouring agents, imposing structural constraints on the control input space. A distributed control-estimation synthesis is proposed which circumvents this issue by integrating distributed estimation for each agent into distributed control law. Based on the agents' state estimate information, the distributed control law allows each agent to interact with non-neighbouring agents, thereby relaxing the structural constraint. Then, the primal optimal distributed control problem is recast to the joint distributed control-estimation problem whose solution can be obtained through the iterative optimization procedure. The stability of the proposed method is verified and the practical effectiveness is supported by numerical simulations and real-world experiments with multi-quadrotor formation flight.

1 | INTRODUCTION

Multi-agent systems (MASs) have gained substantial attention due to their applications in various domains, such as surveillance [1], formation of mobile vehicles [2], production process management [3], power grids [4], etc. These systems involve interconnected sub-components collaborating to achieve global objectives [5–7]. One of the key enabling techniques for MASs is distributed control, emphasizing interactions among neighbouring agents to coordinate with each other instead of relying on a central supervisory control [6, 8–10].

While extensive research has explored different variants of distributed control problems, including communication delays [11, 12], data transmission overhead [13], uncertain agent dynamics [14–18], and stochastic characteristics [19], the quest for an optimal distributed control law for MAS remains unresolved. Especially, the network topological constraint limits the information available to each agent, making it challenging to design optimal control laws. It is worth noting that optimal control laws subject to such constraints may not be linear even for linear MAS dynamics under Gaussian noise and quadratic cost, and that the optimal distributed control problem may be non-convex [20].

To ease this challenge, some have attempted to find optimal solutions tailored to specific classes of MAS control problems in terms of cost functions [21–25], network topology [26–29], and system dynamics [30, 31]. However, these approaches lack applicability to general MAS control problems, especially when dealing with predefined global costs or network topological constraints that do not conform to the specifications for finding tractable optimal solutions. Alternative approaches have been proposed with the goal of deriving suboptimal solutions [32–41]. In addition, certain researchers have chosen to focus solely on individual agents' local cost functions, without considering the global objective [42–45]. Consequently, these approaches compromise the global performance of the entire MAS.

Another line of work has pivoted towards reformulating the optimal distributed control problem into a convex optimization problem [46–48]. Various parameterization techniques, such as Youla [49], system-level [50], and input–output [51] methods, have been studied to accommodate the topological constraint within the convex formulation. Notably, the quadratic invariance (QI) condition has been established, under which the problem can be formulated as a convex problem [52, 53]. Nonetheless, the QI condition restricts the types of solvable problems, leaving the non-QI MAS cases difficult to address. To tackle the broader class of problems beyond the QI cases, various conditions have been established to enable tractable solutions for non-QI cases [54, 55], or approximation strategies have been developed to relax the condition [47, 48]. Despite all these attempts, navigating the complexities of the general class of MAS problem remains a formidable challenge.

To circumvent the network topological constraints, other approaches suggest utilizing distributed estimators to augment information access for individual agents. This empowers agents to estimate beyond their neighbours, thereby enabling distributed control based on the estimated information. Several distributed estimators have been employed for this idea, but they have not adequately addressed the optimality of distributed control laws [56–58]. Tackling the optimal distributed estimation-based control, [59] has leveraged information-sharing via network communication, based on which optimizes the control law referring to the state estimates from a series of intermediate communication steps. As the number of intermediate communication steps increases, the resulting control law gradually converges towards the optimal solution of a centralized MAS control problem without any network topological constraints, but at the cost of excessive communication overhead. On the other hand, [60] has explored the multi-hop communication scheme, and addressed the necessary multi-hop conditions whereby the estimation and control problems can be conceived independently. However, applicability diminishes when communication fails to meet these conditions, leading to intertwined and inseparable distributed estimation-based control problems. Hence, even with the aid of distributed estimation, the optimal distributed control in general MAS remains an open question to date.

This paper elaborates on the distributed estimation-based control to fulfill the quadratic cost for stochastic linear MAS over an infinite-time horizon. Unlike earlier approaches in distributed estimation-based control that separate the estimation and control problems [59, 60], our work tackles an intertwined control-estimation problem without imposing specific conditions on network topology or MAS dynamics, extending beyond existing works that are only valid for specific network structures or predefined cost functions [21–31]. Considering the fact that the design of the distributed estimator is influenced by the distributed control design and vice versa, an iterative optimization procedure is introduced to design distributed linear control and estimation laws. Notably, the proposed method only requires individual agents to measure their immediate neighbours, without relying on additional communication capabilities, such as multi-intermediate or multi-hop communications. This significantly reduces the communication overhead and is beneficial in environments with limited or restricted communication, improving practicality and efficiency in real-world MAS operations.

It is noted that this study builds upon our prior works [61–63]. Initially, we have introduced a distributed estimator that augments its estimation range beyond its neighbour based on distributed control law information [61]. Subsequently, in [62], we have developed distributed control laws that utilize non-neighbouring estimation information, although the intertwined nature of control and estimator design was not addressed. Lastly, [63] has sought to jointly design distributed control and estimation laws. However, this work merely provided the methodological idea without theoretical rigor and lacked practical efficacy, since the solution is given for the finite horizon problem. Motivated by these limits, this paper further enhances the distributed control and estimation laws by addressing the infinite horizon solution along with the theoretical stability guarantee. The key contributions of this paper include the following:

1. Transforming the optimal distributed control problem into a distributed control-estimation synthesis problem for stochastic linear MAS under limited network connectivity.
2. Developing an iterative optimization framework to design steady-state gains for the distributed control and distributed estimator over an infinite horizon.
3. Theoretically verifying the stability of the proposed distributed control and estimation laws.
4. Demonstrating the effectiveness and practical virtue of the proposed method by conducting Monte Carlo simulations and real-world experiments involving multi-quadrotor formation flight.

The rest of the paper is structured as follows: Section 2 formulates the infinite-horizon distributed control-estimation problem based on the dynamical MAS model and the cost function to be optimized. Section 3 develops the main algorithm to optimize the control and estimator gains for each agent along with the theoretic guarantee for its stability. The numerical simulation and the real-world experiment of the proposed method are presented with the multi-quadrotor formation flight in Sections 4 and 5, respectively. Conclusions and future works are given in Section 6.

Notation. The set of real and natural numbers are denoted as $\mathbb{R}$, $\mathbb{N}$, respectively. The expectation of the random variable x is represented as $\mathbb{E}[x]$. Conditional expectation of x given y is represented as $\mathbb{E}[x|y]$. The Kronecker product of matrices is symbolized by $\otimes$. An identity matrix of size $p \times p$ and a zero matrix of the same size are represented as I_p and 0_p, respectively. A vector with all entries equal to 1 is written as $1_p \in \mathbb{R}^p$. $M_i = [0_p \cdots I_p \cdots 0_p] \in \mathbb{R}^{p \times Np}$ denotes the block matrix that contains an i^{th} block filled with I_p, while other block entries are set to 0_p. $blkdg(D_1, \ldots, D_n)$ and $diag(d_1, \ldots, d_n)$ respectively represent a block-diagonal matrix with blocks $D_1, \ldots, D_n$ and a diagonal matrix with entries $d_1, \ldots, d_n$. The trace of a matrix D is denoted as $Tr(D)$. For a symmetric matrix $X = X^{\mathrm{T}}$, we use $X \succ 0$ and $X \succeq 0$ to denote positive definite and positive semidefinite, respectively. $\|\cdot\|_F$ represent the Frobenius norm. The spectrum of a matrix $\bullet$ is denoted as $spec(\bullet)$. The null space of a matrix $\bullet$ is represented as $\mathcal{N}(\bullet)$. $(S)^{\perp}$ signifies the orthogonal complement of set S.

2 | PROBLEM FORMULATION

2.1 | Stochastic multi-agent system dynamics model

We consider a MAS comprised of N homogeneous agents, where the stochastic linear time-invariant dynamics of an individual agent is described by

$$x_i(k+1) = Ax_i(k) + Bu_i(k) + w_i(k), \forall i \in \{1, \ldots, N\}, \quad (1)$$

where $x_i \in \mathbb{R}^n$ and $u_i \in \mathbb{R}^p$ are state and the control input of the i^{th} agent, respectively. The disturbance in the dynamics, w_i, is modelled by the Gaussian noise with zero mean and covariance matrix $\Theta_i \succ 0$. $k \in \mathbb{N}$ is discrete-time index. Without loss of generality, the matrix pair (A, B) satisfies the controllability condition. By concatenating the states of individual agents, the dynamics of the entire MAS can be represented by

$$x(k+1) = \tilde{A}x(k) + \tilde{B}u(k) + \tilde{w}(k), \quad (2)$$

$$\tilde{A} = (I_N \otimes A),\ \tilde{B} = (I_N \otimes B),$$

$$x(k) = \left[x_1^{\mathrm{T}}(k) \cdots x_N^{\mathrm{T}}(k)\right]^{\mathrm{T}},\ u(k) = \left[u_1^{\mathrm{T}}(k) \cdots u_N^{\mathrm{T}}(k)\right]^{\mathrm{T}}$$

$$\tilde{w}(k) = \left[w_1^{\mathrm{T}}(k) \cdots w_N^{\mathrm{T}}(k)\right]^{\mathrm{T}}.$$

Apart from the MAS dynamics, network topology is represented as a directed graph model [5]. Agents are vertices in the set $\mathcal{V} = \{1, 2, \ldots, N\}$, connected by edges $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$. The adjacency matrix $\mathcal{A} = [a_{ij}] \in \mathbb{R}^{N \times N}$ denotes the entire network connectivity, where $a_{ij} = 1$ if $(i, j) \in \mathcal{E}$, and $a_{ij} = 0$ otherwise. We also define the degree matrix $\mathcal{D} = diag(\sum_j a_{1j}, \ldots, \sum_j a_{Nj})$ and the Laplacian matrix $\mathcal{L} = \mathcal{D} - \mathcal{A}$. The neighbours of the i^{th} agent are denoted as a set Ω_i, and its cardinality is represented by $|\Omega_i|$. With the given network connection, the i^{th} agent obtains noisy measurements from its neighbours as follows [61]:

$$z_{ij}(k) = a_{ij}\left(x_j(k) + v_{ij}(k)\right), \quad \forall j \in \mathcal{V}, \quad (3)$$

where v_{ij} is white Gaussian noise with covariance $\Xi_{ij} \succ 0$. The coefficient a_{ij} in (3) indicates the availability of the measurement of the j^{th} agent's state from the i^{th} agent's perspective. The noisy measurement z_{ij} can be obtained either through on-board sensors employed by the agents or, alternatively, via communication. For example, in robotic applications, each agent can transmit its position measurement to neighbouring agents through communication, while on-board radar sensors measure the velocity of neighbouring agents. Both measurements are susceptible to errors due to environmental uncertainties or inherent sensor quality, and these errors are reflected in the noise term v_{ij}. Besides, the set of measurements and noises acquired by the i^{th} agent are denoted by $Z_i(k) = \left[z_{i1}^{\mathrm{T}}(k) \cdots z_{iN}^{\mathrm{T}}(k)\right]^{\mathrm{T}}$ and $v_i(k) = \left[v_{i1}^{\mathrm{T}}(k) \cdots v_{iN}^{\mathrm{T}}(k)\right]^{\mathrm{T}}$, respectively. It is noted that Z_i is a concatenation of z_{ij}, $j \in \mathcal{V}$, measuring all agents' states in the MAS from the i^{th} agent's perspective. Therefore, only the measurements for neighbouring agents have valid values, while those for non-neighbouring agents are set to zeros. Then, agents interact with their local neighbourhoods based on the following output feedback control law [38]:

$$u_i(k) = M_i F Z_i(k),\ \forall i \in \mathcal{V}, \quad (4)$$

where $F \in \mathbb{F}$ represents the steady-state control gain, and subspace $\mathbb{F} \subseteq \mathbb{R}^{Np \times Nn}$ represents the structural constraints enforced by the MAS's network topology. Note that the feedback control law (4) forms in a linear structure, necessitating more system conditions to grant the global optimum [64]. In this paper, we confine our examination to this linear structure due to its extensive utilization in distributed control [37, 59].

2.2 | Distributed control-estimation synthesis problem

The optimal distributed control problem for the stochastic MAS model can be formulated as follows.

Problem 1. Infinite-horizon optimal distributed linear control law subject to network topological constraint.

$$\min_{F \in \mathbb{F}} \lim_{T \to \infty} \frac{1}{T} \sum_{k=0}^{T} \mathbb{E}\left[x^{\mathrm{T}}(k)Qx(k) + u^{\mathrm{T}}(k)Ru(k)\right]$$

$$\text{subject to (2), and (4), } \forall i \in \mathcal{V}$$

where $Q \in \mathbb{R}^{Nn \times Nn} \succeq 0$ is a weight matrix that governs the relative state coordination between agents. It can be used to measure the state differences between neighbouring agents, or extended to capture the differences between non-neighbouring agents as well. $R \in \mathbb{R}^{Np \times Np} \succ 0$ is the associated weight matrix used to scale the input cost. This quadratic cost function is commonly employed to represent various inter-agent behaviours in MAS, including consensus [65], formation [2], rendezvous [66], and so on.

Problem 1 is subjected to topological constraints that embed the structural constraints into the gain matrix subspace $\mathbb{F}$, making the problem non-convex [36]. To address the complexity due to topological constraints, we employ the concept of 'virtual interaction'. This employs the estimation-based feedback control, facilitated by the distributed estimator proposed in [61]. The key idea is to allow agents without network connections to act as if they could share information and interact with each other. Subsequently, the concept of a 'virtual network topology' is introduced to describe the connections formed through these virtual interactions, regardless of the actual network topology in MAS (See fig. 1 of [63]). Leveraging the proposed distributed estimators, individual agents can estimate the entire MAS state, including agents not connected through the actual network topology, using only neighbouring measurements. These estimates, incorporating both neighbouring and non-neighbouring agents' states, allow each agent to apply

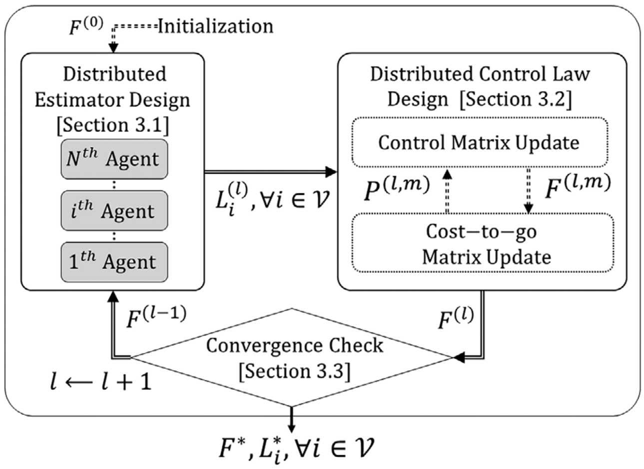

FIGURE 1 Control-estimation laws design procedure.

estimation-based control. This process enables agents to interact beyond their immediate neighbours by utilizing the estimates of non-neighbouring agents' states, which we refer to as virtual interactions. Accordingly, the virtual network topology refers to the connections established by these virtual interactions between agents. This demonstrates that the virtual network topology is not a topological constraint but rather a design choice.

Definition 1. Given the distributed estimator embedded in each agent, the state estimate of the MAS from the i^{th} agent's perspective is denoted as ${}^{i}\hat{x}(k) := \mathbb{E}\left[x(k)|Z_{i,(0:k)}\right]$, where $Z_{i,(0:k)} = \{Z_i(t)| \quad t = 0, \dots, k\}$ represents measurements up to time step k. The corresponding estimation error covariance is represented as ${}^{ii}\Sigma(k) := \mathbb{E}\left[{}^{i}e(k)\,{}^{i}e(k)^{\mathrm{T}}|Z_{i,(0:k)}\right]$, where ${}^{i}e(k) := x(k) - {}^{i}\hat{x}(k)$ is the estimation error. Additionally, the predicted state estimate and its estimation error covariance at the next time step are denoted as ${}^{i}\hat{x}^{-}(k+1) := \mathbb{E}\left[x(k+1)|Z_{i,(0:k)}\right]$ and ${}^{ii}\Sigma^{-}(k+1) := \mathbb{E}\left[{}^{i}e^{-}(k+1)\,{}^{i}e^{-}(k+1)^{\mathrm{T}}|Z_{i,(0:k)}\right]$, respectively, where ${}^{i}e^{-}(k+1) := x(k+1) - {}^{i}\hat{x}^{-}(k+1)$. Furthermore, the concatenated estimation error from every agent's perspective and its covariance are defined as $e(k) = [{}^{1}e(k)^{\mathrm{T}} \cdots {}^{N}e(k)^{\mathrm{T}}]^{\mathrm{T}} \in \mathbb{R}^{NnN}$ and $\Sigma(k) := \mathbb{E}[e(k)e(k)^{\mathrm{T}}|Z_{i,(0:k)}, \forall i \in \mathcal{V}] \in \mathbb{R}^{NnN\times NnN}$, respectively. The predicted estimation errors from every agent's perspective $e^{-}(k)$ and its covariance $\Sigma^{-}(k)$ are defined in the same manner.

Acquiring the measurement $Z_i(k)$ at each time step, the state estimate of the MAS from the i^{th} agent's perspective is recursively updated by the following linear estimator with the estimator gain, $L_i \in \mathbb{R}^{nN\times n|\Omega_i|}$,

$$ {}^{i}\hat{x}(k) = {}^{i}\hat{x}^{-}(k) + L_i\left(H_i Z_i(k) - H_i\,{}^{i}\hat{x}^{-}(k)\right), \tag{5} $$

where $H_i = [h_1 \ h_2 \ \cdots h_{|\Omega_i|}]^{\mathrm{T}} \otimes I_n \in \mathbb{R}^{n|\Omega_i|\times nN}$ filters out the MAS state entries of non-neighbouring agents of the i^{th} agent where $h_{m=1,2,\dots,|\Omega_i|} \in \mathbb{R}^{N}$ are the non-zero column vectors of the matrix $diag(a_{i1}, a_{i2} \dots, a_{iN})$. It is important to note that the innovation term in (5), denoted as $\left(H_i Z_i(k) - H_i\,{}^{i}\hat{x}^{-}(k)\right)$, solely captures the measurements of neighbouring agents respecting the network topology [63], and each agent uses this residual information to update its estimates of the entire MAS state.

With the help of the estimated states, each agent can implement the estimation-based feedback control law, realizing virtual interactions not only with neighbouring but also with non-neighbouring agents. Accordingly, we modify (4) to the distributed estimation-based feedback control law as follows:

$$ u_i(k) = M_i F\,{}^{i}\hat{x}(k), \ \forall i \in \mathcal{V}. \tag{6} $$

In (6), the subspace of the control gain F is no longer constrained by the actual network topology since the distributed estimator embedded in each agent provides access to the states of non-neighbouring agents. With the inclusion of the estimation-based control law, we proceed to recast the original distributed control Problem 1 into a unified distributed control-estimation problem. This allows us to simultaneously optimize both the control and estimation laws as follows.

Problem 2. Virtual interaction-based optimal distributed linear control-estimation laws.

$$ \min_{F\in\tilde{\mathbb{F}}, L_i, \forall i\in\mathcal{V}} J(F, L_1, \dots, L_N) $$

$$ \text{subject to (2), (5), and (6), } \forall i \in \mathcal{V} $$

where $J(F, L_1, \dots, L_N) := \lim_{T\to\infty} \frac{1}{T} \sum_{k=0}^{T} \mathbb{E}[x^{\mathrm{T}}(k)Qx(k) + u^{\mathrm{T}}(k)\,Ru(k)]$.

Remark 1. In contrast to $\mathbb{F}$, the subspace $\tilde{\mathbb{F}}$ is a constraint-free space since individual agents can access the estimate of the entire system state and interact beyond their neighbours despite the constraint $\mathbb{F}$. Thus, no structural constraints are placed on the matrix F in (6), offering full flexibility in designing the virtual network topology. This means that the virtual network topology is not dictated by the physical structure of the network but is instead a design choice. Without loss of generality, this paper adopts the entire matrix space, that is, $\tilde{\mathbb{F}} = \mathbb{R}^{Np\times Nn}$, invoking a fully connected virtual network topology that removes structural constraints in virtual interactions. Exploring the design of virtual network topology is an interesting topic but beyond the scope of this paper and will be investigated in future work.

Unlike centralized systems, where the separation principle can be applied to derive the optimal control and estimator gains separately, the separation principle is not viable in distributed MASs due to the mutual interdependence between the controller and estimator [63]. Consequently, this renders Problem 2 inherently non-convex that often requires strict assumptions for a globally optimal solution [21–25, 46–48]. To address this issue, we propose a framework that iteratively optimizes control and estimator gains for each agent without imposing stringent constraints on the problem formulation.

Specifically, as illustrated in Figure 1, our method involves two sequential design steps at each iteration (denoted as l^{th}). First, in the *distributed estimator design*, the estimator gains, denoted as $L_i^{(l)}, \forall i \in \mathcal{V}$, are computed (Section 3.1). These gains are determined based on the distributed control gains calculated from the previous iteration. Subsequently, in the *distributed control law design*, the optimization of the control gain $F^{(l)}$ takes place (Section 3.2). This optimization considers the state estimation errors induced by the distributed estimators from the preceding *distributed estimator design*. Then the control and estimator gains obtained at the current iteration undergo a convergence check and are then applied to the next iteration (Section 3.3). This iterative optimization process continues until predefined stopping criteria are met, returning the final control and estimator gains represented by F^* and $L_i^*, \forall i \in \mathcal{V}$. It is worth noting that the iterative optimization occurs during the offline design phase, utilizing information from the entire MAS in a centralized manner. This design phase contrasts with the actual operation of the MAS, which is executed in a distributed fashion based on the following assumptions.

Assumption 1. Upon deployment, each i^{th} agent in MAS is equipped with the system dynamics, distributed control, and estimator gains, that is, A, B, F^*, and L_i^*. This prior information enables the individual agents to manage state estimation and control during distributed MAS operations.

3 | ALGORITHM DEVELOPMENT

This section outlines the procedure of the proposed control-estimation synthesis for distributed MAS.

3.1 | Distributed estimator design

The aim of the distributed estimator design is to optimize the distributed estimator gains for each agent, denoted as $L_i^{(l)}, \forall i \in \mathcal{V}$ that minimize the estimation error covariances for each agent, ${}^{ii}\Sigma, \forall i \in \mathcal{V}$. The foundation for the distributed estimator design stems from [61], but this subsection extends it to compute the steady-state gains suitable for addressing infinite-time horizon scenarios. During this offline design process, we exploit information such as agent dynamics (A, B), neighbouring agents (Ω_i), and the control law designed in the previous iteration, denoted as $F^{(l-1)}$ (abbreviated as F in this subsection for brevity).

By applying (6) with Definition 1, one can rewrite the dynamics of MAS (2) as follows:

$$\begin{aligned} x(k+1) &= (\tilde{A} + \tilde{B}F)x(k) - \sum_{i}^{N} \tilde{B}\bar{M}_i F\, {}^{i}e(k) + \tilde{w}(k) \\ &= (\tilde{A} + \tilde{B}F)x(k) - \Pi e(k) + \tilde{w}(k), \end{aligned} \tag{7}$$

where $\Pi = \tilde{\tilde{B}}\tilde{M}\tilde{F}$, $\tilde{\tilde{B}} = 1_N^{\mathrm{T}} \otimes \tilde{B}$, $\tilde{M} = blkdg(\bar{M}_1, \dots, \bar{M}_N)$, $\bar{M}_i = M_i^{\mathrm{T}} M_i, \forall i \in \mathcal{V}$, and $\tilde{F} = I_N \otimes F$. The predicted state estimate of the MAS, as viewed from the perspective of the i^{th} agent, can be expressed as follows:

$${}^{i}\hat{x}^{-}(k+1) = (\tilde{A} + \tilde{B}F)\, {}^{i}\hat{x}(k). \tag{8}$$

Then ${}^{i}e^{-}(k+1)$ can be computed by subtracting (8) from (7). Furthermore, ${}^{i}e^{-}(k+1), \forall i \in \mathcal{V}$ are concatenated to represent the predicted estimation error and its covariance as follows:

$$\begin{aligned} e^{-}(k+1) &= \Lambda_k e(k) + 1_N \otimes \tilde{w}(k), \\ \Sigma^{-}(k+1) &= \Lambda_k \Sigma(k) \Lambda_k^{\mathrm{T}} + \Sigma_{\tilde{w}}, \end{aligned} \tag{9}$$

where $\Lambda_k = I_N \otimes (\tilde{A} + \tilde{B}F) - 1_N \otimes \tilde{\tilde{B}}\tilde{M}\tilde{F}$, and $\Sigma_{\tilde{w}} = (1_N 1_N^{\mathrm{T}}) \otimes blkdg(\Theta_1, \dots, \Theta_N)$. The diagonal block-matrices in $\Sigma^{-}(k+1)$ represent the predicted estimation error covariance from the perspective of each agent, denoted as ${}^{ii}\Sigma^{-}(k+1), \forall i \in \mathcal{V}$, while the off-diagonal block-matrices indicate cross-covariances between the predicted state estimates of two different agents, that is, ${}^{ij}\Sigma^{-}(k+1) = \mathbb{E}[{}^{i}e^{-}(k+1)\, {}^{j}e^{-}(k+1)^{\mathrm{T}}], \forall i \neq j \in \mathcal{V}$.

Following the recursive update form (5), the predicted state estimate is updated with the estimator gain $L_i(k+1)$. This estimator gain is designed to minimize the trace of the updated estimation error covariance, i.e., $\mathrm{Tr}\left({}^{ii}\Sigma(k+1)\right)$, which yields,

$$L_i(k+1) = {}^{ii}\Sigma^{-}(k+1) H_i^{\mathrm{T}} (S_i(k+1))^{-1}, \tag{10}$$

where,

$$\begin{aligned} S_i(k+1) &= H_i({}^{ii}\Sigma^{-}(k+1) + {}^{i}\Xi) H_i^{\mathrm{T}}, \\ {}^{i}\Xi &= blkdg(\Xi_{i1},\ \Xi_{i2}, \dots \Xi_{iN}). \end{aligned}$$

Subsequently, the estimation error $e(k+1)$ and its covariance $\Sigma(k+1)$ are updated as follows:

$$\begin{aligned} e(k+1) &= (I - \tilde{L}(k+1))e^{-}(k+1) + \tilde{L}(k+1)\tilde{V}(k+1), \\ \Sigma(k+1) &= (I - \tilde{L}(k+1))\Sigma^{-}(k+1)(I - \tilde{L}(k+1))^{\mathrm{T}} \\ &\quad + \tilde{L}(k+1)\Sigma_{\Xi}\tilde{L}(k+1)^{\mathrm{T}}, \end{aligned} \tag{11}$$

where $\tilde{V}(k+1) = \left[v_1^{\mathrm{T}}(k+1) \cdots v_N^{\mathrm{T}}(k+1)\right]^{\mathrm{T}}$, $\tilde{L}(k+1) = blkdg(L_1(k+1)H_1, \dots, L_N(k+1)H_N)$, and $\Sigma_{\Xi} = blkdg({}^{1}\Xi, \dots, {}^{N}\Xi)$. Starting from the initial error covariance $\Sigma(0)$, we update the error covariance through equations (9), (10), and (11) at each time step $k = 0, 1, \dots$. This continues until the estimation error covariance converges to steady-state, as determined by the following criterion,

$$D_{KL}\left(p(k) || p(k-1)\right) < \tilde{\epsilon}_{est}, \tag{12}$$

where D_{KL} is the Kullback–Leibler (KL) divergence between estimation error statistics over the consecutive time steps, $p(k) \sim \mathcal{N}\left(0_{nNN}, \Sigma(k)\right)$, and $p(k-1) \sim \mathcal{N}(0_{nNN}, \Sigma(k-1))$. KL divergence is used for its ability to quantify differences between probability distributions and its computational efficiency [67]. Additionally, $\tilde{\epsilon}_{est}$ denotes the pre-set stopping tolerance. Once (12) is satisfied at $\tilde{k}$ time step, the steady-state estimator gains and the corresponding estimation error covariances are respectively determined as $L_i^{(l)} := L_i(\tilde{k}), \forall i \in \mathcal{V}$, and $\Sigma_e^{(l)} := \Sigma(\tilde{k})$, and the update terminates. This ends the l^{th} design iteration of the distributed estimator gains given the distributed control gain $F = F^{(l-1)}$. The computed $L_i^{(l)}, \forall i \in \mathcal{V}$ and $\Sigma_e^{(l)}$ will be utilized in the subsequent stage for the design of the distributed control gain. Please refer to our prior studies [61–63] for the detailed derivation and stability analysis of the proposed distributed estimator.

Remark 2. The derived distributed estimator processes neighbouring measurements only, yet each agent can estimate the states of the entire MAS, including non-neighbouring agents.

3.2 | Distributed control law design

In this subsection, the design process for the distributed control gain is developed based on the embedded distributed estimator within individual agents. The estimator gains and the associated estimation error covariance computed in the previous subsection, that is, $L_i^{(l)}$ and $\Sigma_e^{(l)}$, are respectively abbreviated as L_i and Σ_e for brevity. Similarly, the control gain, which serves as the design parameter in this subsection, is denoted as $F^{(l)}$ and will be abbreviated as F. The optimization of F in the l^{th} iteration is then formulated as follows.

Problem 3. Infinite-horizon optimal distributed linear control law design with the fixed distributed estimation law, $L_i, \forall i \in \mathcal{V}$,

$$\min_{F\in\mathbb{F}} \lim_{T\to\infty} \frac{1}{T}\sum_{k=0}^{T} \mathbb{E}[x^{\mathrm{T}}(k)Qx(k) + u^{\mathrm{T}}(k)Ru(k)]$$

subject to (2), (5), and (6)

In light of dynamic programming, let us first define the optimal cost-to-go function as $V_k(Z_{(0:k)})$ where $Z_{(0:k)} = \{z_{i,(0:k)}, \forall i \in \mathcal{V}\}$ is the collection of all measurements each acquired by individual agents up to time step k. Then, from the Bellman's optimality principle [68], the optimal cost-to-go can be represented as follows:

$$\begin{aligned} V_k(Z_{(0:k)}) = \min_{F\in\mathbb{F}} \mathbb{E}\left[x^{\mathrm{T}}(k)Qx(k) + u^{\mathrm{T}}(k)Ru(k)\right. \\ \left. + V_{k+1}(Z_{(0:k+1)})|Z_{(0:k)}\right]. \end{aligned} \tag{13}$$

To address the expectation presented in (13), we introduce the notation $\bar{x}(k) = \mathbb{E}[x(k)|Z_{(0:k)}]$ to represent the state expectation (or equivalently state estimate) conditioned on the entire measurement set of the MAS up to time step k. Similarly, the covariance of this expected estimate is denoted as $\Sigma_{\bar{x}}(k) = \mathbb{E}[(x(k) - \mathbb{E}[x(k)|Z_{(0:k)}])(x(k) - \mathbb{E}[x(k)|Z_{(0:k)}])^{\mathrm{T}}]$. Notably, this state expectation is estimated centrally and is only utilized for the offline design phase, not accessible to individual agents during the online operational phase. Given the initial covariance $\Sigma_{\bar{x}}(0)$ and utilizing (7), the covariance of the expected states can be recursively updated as follows:

$$\Sigma_{\bar{x}}(k+1) = (\tilde{A} + \tilde{B}F)\Sigma_{\bar{x}}(k)(\tilde{A} + \tilde{B}F)^{\mathrm{T}} + \Pi\Sigma(k)\Pi^{\mathrm{T}} + \Sigma_{\tilde{w}}.$$

When the estimation error covariance converges to a steady-state, that is, $\Sigma(k) \approx \Sigma_e$, the corresponding steady-state covariance of the expected state, denoted as $\Sigma_{\bar{x}}$, is given by,

$$\Sigma_{\bar{x}} = (\tilde{A} + \tilde{B}F)\Sigma_{\bar{x}}(\tilde{A} + \tilde{B}F)^{\mathrm{T}} + \Pi\Sigma_e\Pi^{\mathrm{T}} + \Sigma_{\tilde{w}}. \tag{14}$$

Using the expected state and its covariance information, the first quadratic term in (13) can be rewritten as follows:

$$\mathbb{E}[x^{\mathrm{T}}(k)Qx(k)|Z_{(0:k)}] = \bar{x}^{\mathrm{T}}(k)Q\bar{x}(k) + Tr(Q\Sigma_{\bar{x}}(k)). \tag{15}$$

Recalling (6), the estimation-based control law can be rewritten in terms of error states as follows:

$$u(k) = Fx(k) - \sum_{i}^{N} \bar{M}_i F\, {}^{i}e(k). \tag{16}$$

Then the second quadratic term in (13) with (16) can be expressed by

$$\begin{aligned} \mathbb{E}[u^{\mathrm{T}}(k)Ru(k)|Z_{(0:k)}] = \sum_{i,j}^{N} Tr(F^{\mathrm{T}}\bar{M}_i^{\mathrm{T}}R\bar{M}_jF\, {}^{ij}\Sigma_e(k)) \\ + \bar{x}^{\mathrm{T}}(k)F^{\mathrm{T}}RF\bar{x}(k) + Tr(F^{\mathrm{T}}RF\Sigma_{\bar{x}}(k)), \end{aligned} \tag{17}$$

where ${}^{ij}\Sigma_e(k) \in \mathbb{R}^{Nn\times Nn}$ represent the block matrices that make up $\Sigma_e(k)$.

Now, let us parameterize the optimal cost-to-go function using the following quadratic form

$$\begin{aligned} V_k\left(Z_{(0:k)}\right) &= \mathbb{E}\left[x(k)^{\mathrm{T}}P(k)x(k)|Z_{(0:k)}\right] + q(k) \\ &= \bar{x}^{\mathrm{T}}(k)P(k)\bar{x}(k) + Tr(P(k)\Sigma_{\bar{x}}(k)) + q(k), \end{aligned} \tag{18}$$

where $P(k)$ is called a cost-to-go matrix and $q(k)$ encompasses the cost terms related to the errors from distributed estimators. Applying (7), the expected $V_{k+1}(Z_{(0:k+1)})$ in (13) can be expressed by

$$\begin{aligned} \mathbb{E}[V_{k+1}(Z_{(0:k+1)})|Z_{(0:k)}] &= Tr(P(k+1)\Sigma_{\tilde{w}}) + q(k+1) \\ &+ \bar{x}^{\mathrm{T}}(k)(\tilde{A} + \tilde{B}F)^{\mathrm{T}}P(k+1)(\tilde{A} + \tilde{B}F)\bar{x}(k) \\ &+ Tr((\tilde{A} + \tilde{B}F)^{\mathrm{T}}P(k+1)(\tilde{A} + \tilde{B}F)\Sigma_{\bar{x}}(k)) \\ &+ Tr(\Pi^{\mathrm{T}}P(k+1)\Pi\Sigma_e(k)). \end{aligned} \tag{19}$$

Then, plugging (15), (17), and (19) into (13) gives

$$
\begin{aligned}
V_k(Z_{(0:k)}) &= \min_{F\in\mathbb{F}} \bar{x}^{\mathrm{T}}(k)\big(Q + F^{\mathrm{T}}RF + (\tilde{A}+\tilde{B}F)^{\mathrm{T}}P(k+1)(\tilde{A}\\
&+\tilde{B}F)\big)\bar{x}(k) + Tr\big((Q + F^{\mathrm{T}}RF + (\tilde{A}+\tilde{B}F)^{\mathrm{T}}P(k+1)\\
&\times(\tilde{A}+\tilde{B}F))\Sigma_{\bar{x}}(k)\big) + \sum_{i,j}^{N} Tr(F^{\mathrm{T}}\bar{M}_i^{\mathrm{T}}R\bar{M}_jF\,{}^{ij}\Sigma_e(k))\\
&+ Tr(\Pi^{\mathrm{T}}P(k+1)\Pi\Sigma_e(k)) + Tr(P(k+1)\Sigma_{\tilde{w}}) + q(k+1).
\end{aligned} \tag{20}
$$

By equating the quadratic terms with respect to $\bar{x}(k)$ from (18) and (20), the optimal cost-to-go matrix $P(k)$ satisfies the following recursion

$$
P(k) = Q + F^{\mathrm{T}}RF + (\tilde{A}+\tilde{B}F)^{\mathrm{T}}P(k+1)(\tilde{A}+\tilde{B}F). \tag{21}
$$

Without loss of generality, the system is assumed to enter the steady-state after sufficiently large k steps. Then, the first term in (20), that is, quadratic cost function with respect to $\bar{x}$, is set to be zero. And the estimation error covariances can be replaced with the steady-state matrices, that is, $\Sigma_{\bar{x}}(k) \approx \Sigma_{\bar{x}}$, and $\Sigma_e(k) \approx \Sigma_e$. Afterward, to find the optimal F, we take the derivative of the remainders in (20) with respect to F and zero them according to the first order optimality condition as follows:

$$
\begin{aligned}
\frac{\partial V_k}{\partial F} &= \sum_{i,j}^{N} \bar{M}_i^{\mathrm{T}}(R + \tilde{B}^{\mathrm{T}}P(k+1)\tilde{B})\bar{M}_jF\,{}^{ij}\Sigma_e\\
&+ (RF + \tilde{B}^{\mathrm{T}}P(k+1)\tilde{A} + \tilde{B}^{\mathrm{T}}P(k+1)\tilde{B}F)\Sigma_{\bar{x}} = 0.
\end{aligned} \tag{22}
$$

In calculating the steady-state solution, the optimal cost-to-go matrix at the l^{th} iteration should be given as a constant matrix $P = P(k) = P(k+1)$. Further, the optimal F should satisfy (22) with this P. To find such a pair of optimal P and F, we alternately solve (22) for F and (21) for P in an iterative manner. That is, F is computed by (22) given P as constant, followed by that P is updated by (21) while F is set to be constant. To this end, let us denote $F^{(l,m)}$ and $P^{(l,m)}$ as the target optimizing variables at the m^{th} iteration of the internal recursion for $F^{(l)}$ and $P^{(l)}$ at the l^{th} iteration of the distributed control law design. Initializing with the optimal cost-to-go matrix in the previous iteration, $P^{(l,0)} = P^{(l-1)}$, a generalized Sylvester equation derived from (22) is solved for computing $F^{(l,m)}$ based on $P^{(l,m)}$ as follows:

$$
\begin{aligned}
&\sum_{i,j}^{N}\bar{M}_i^{\mathrm{T}}(R + \tilde{B}^{\mathrm{T}}P^{(l,m)}\tilde{B})\bar{M}_jF^{(l,m)}\,{}^{ij}\Sigma_e + (RF^{(l,m)}\\
&+ \tilde{B}^{\mathrm{T}}P^{(l,m)}\tilde{A} + \tilde{B}^{\mathrm{T}}P^{(l,m)}\tilde{B}F^{(l,m)})\Sigma_{\bar{x}} = 0.
\end{aligned} \tag{23}
$$

Then, $P^{(l,m)}$ is updated by

$$
\begin{aligned}
P^{(l,m+1)} &= Q + F^{(l,m)\mathrm{T}}RF^{(l,m)} + \tilde{A}^{\mathrm{T}}P^{(l,m)}\tilde{A}\\
&+ \tilde{A}^{\mathrm{T}}P^{(l,m)}\tilde{B}F^{(l,m)} + F^{(l,m)\mathrm{T}}\tilde{B}^{\mathrm{T}}P^{(l,m)}\tilde{A}\\
&+ F^{(l,m)\mathrm{T}}\tilde{B}^{\mathrm{T}}P^{(l,m)}\tilde{B}F^{(l,m)}.
\end{aligned} \tag{24}
$$

Equations (23) and (24) are repeated until the cost-to-go matrix $P^{(l,m)}$ satisfies the stopping condition as follows:

$$
\| P^{(l,m)} - P^{(l,m-1)} \|_F < \tilde{\epsilon}_P, \tag{25}
$$

where the stopping threshold $\tilde{\epsilon}_P$ is set sufficiently small to ensure that the computed $P^{(l,m)}$ adheres to the conditions in (21). Once the iteration terminates at η step, we set $F^{(l)} = F^{(l,\eta)}, P^{(l)} = P^{(l,\eta)}$ as the designed steady-state control gain and the respective cost-to-go matrix at the l^{th} iteration, respectively.

Remark 3. In the ideal case of a MAS with negligible estimation errors, that is, ${}^{ij}\Sigma_e \approx 0, \forall i,j \in \mathcal{V}$, the first term in (22) becomes negligible, resulting in

$$
\frac{\partial V_k}{\partial F} = (RF + \tilde{B}^{\mathrm{T}}P(k+1)\tilde{A} + \tilde{B}^{\mathrm{T}}P(k+1)\tilde{B}F)\Sigma_{\bar{x}} = 0.
$$

Then, the optimal F can be analytically derived as follows:

$$
F = -(\tilde{B}^{\mathrm{T}}P(k+1)\tilde{B} + R)^{-1}\tilde{B}^{\mathrm{T}}P(k+1)\tilde{A}.
$$

This result coincides with the centralized LQR solution. And it is well matched to the fact that, without estimation errors, individual agents have access to the full MAS state information, and the original distributed control problem becomes a centralized LQR problem.

3.3 | Convergence check

After optimizing the distributed control and estimation laws at the current iteration, we assess the computed $F^{(l)}$ and $L_i^{(l)}, \forall i \in \mathcal{V}$ to decide whether to advance to the next iteration or not. Throughout the iteration, a group of optimized variables is stored in the following set

$$
\tilde{\mathfrak{I}} := \left\{ \tilde{\mathfrak{z}}^{(l)} \mid \tilde{\mathfrak{z}}^{(l)} = \left(F^{(l)}, P^{(l)}, L_1^{(l)}, \ldots, L_N^{(l)}, \Sigma_e^{(l)}\right), \forall l \in \mathbb{N} \right\}.
$$

The iteration of the optimizations concludes when either: (i) the maximum specified number of iterations, $\tilde{N}_{\max}$, is reached, or (ii) the consecutive iterations satisfy the following stopping condition,

$$
|\Delta^{(l)} - \Delta^{(l-1)}| \le \tilde{\epsilon}_{\text{stop}}, \tag{26}
$$

ALGORITHM 1 (Design phase) Virtual interaction-based distributed linear control-estimation synthesis

Initialization
- Set the MAS dynamics information A, B, $\mathcal{L}$, $\Sigma_{\tilde{w}}$, Σ_{Ξ}, and the cost metrics Q, R.
- Initialize $F^{(0)}$, and $P^{(0)}$.
- Set stopping thresholds $\tilde{\epsilon}_{est}$, $\tilde{\epsilon}_P$, $\tilde{\epsilon}_{\text{stop}}$ sufficiently small, and the maximum number of iteration, $\tilde{N}_{\max}$.

$l = 0$

While $l \le \tilde{N}_{\max}$

(a) **Distributed estimator design**
  While (12) is not satisfied
  (1) Compute $L_i(k), \forall i \in \mathcal{V}$, and $\Sigma(k)$ using $F^{(l)}$, (9), (10), and (11).
  $k \leftarrow k + 1$
  end While
  Output $\Longrightarrow L_i^{(l)}, \forall i \in \mathcal{V}$, and $\Sigma_e^{(l)}$

(b) **Distributed control law design**
  (2) Set $P^{(l,0)} = P^{(l-1)}$
  (3) Compute $\Sigma_{\tilde{x}}$, by solving (14).
  While (25) is not satisfied
  (3) Compute $F^{(l,m)}$ by solving (23).
  (4) Update $P^{(l,m+1)}$ using (24).
  $m \leftarrow m + 1$
  end While
  Output $\Longrightarrow F^{(l)}$

(c) **Convergence check**
  (5) Store $F^{(l)}, P^{(l)}, \Sigma_e^{(l)}, L_i^{(l)}, \forall i \in \mathcal{V}$ in the set $\tilde{S}$.
  (6) **If** (26) is satisfied $\longrightarrow$ **Break**
  else $l \leftarrow l + 1$

end while

Return $\Longrightarrow F^*$, and $L_i^*, \forall i \in \mathcal{V}$ using (27)

where the threshold $\tilde{\epsilon}_{\text{stop}}$ is set sufficiently small, and $\Delta^{(l)}$ is the average stage cost defined by,

$$\Delta^{(l)} := Tr(\Pi^{(l)\mathrm{T}} P^{(l)} \Pi^{(l)} \Sigma_e^{(l)}) + Tr(P^{(l)} \Sigma_{\tilde{w}}) + \sum_{i,j}^{N} Tr(F^{(l)\mathrm{T}} \bar{M}_i^{\mathrm{T}} R \bar{M}_j F^{(l)} \, {}^{ij}\Sigma_e^{(l)}),$$

and $\Pi^{(l)} = \tilde{\bar{B}} \tilde{M} \tilde{F}^{(l)}$. Once the iteration ends, the resulting control and estimator gains are denoted by,

$$F^* = F^{(\tilde{\kappa})}, \; L_i^* = L_i^{(\tilde{\kappa})}, \forall i \in \mathcal{V} \quad \text{where } \tilde{\kappa} = \arg \min_{\forall l \in |\tilde{S}|} \Delta^{(l)}. \tag{27}$$

The overall recursive structure of the design procedure is summarized in Algorithm 1. During the operational phase, individual agents employ F^* and L_i^* to execute the online operation of the MAS, as detailed in Algorithm 2.

Remark 4. The proposed distributed control-estimation synthesis occurs during the offline design phase before deployment of the MAS to online operation. In the design phase (Algorithm 1), the primary computational complexity, which affects the scalability of the algorithm in relation to the number of states and agents in the MAS, arises from solving the matrix equation in (23) and calculating the inverse matrix $(S_i(k+1))^{-1}$ in (10), with quantifiable computational complexity orders of $\mathcal{O}((Nn)^2)$ and $\mathcal{O}((Nn)^3)$, respectively. For online operations (Algorithm 2), the onboard computation of each agent involves only elemental matrix and vector multiplications, as presented in (2), (3), (5), (6), and (8). Thus, online computation remains viable for individual agents' limited computing resources.

ALGORITHM 2 (Operational phase) Virtual interaction based distributed linear control for the i^{th} agent

Initialization
- Set the MAS dynamics information A, B, H_i, the initial condition ${}^i\hat{x}(0)$, the optimized control and estimator gains (F^*, L_i^*).

$k = 0$

While $k \ge 0$ do
(1) Execute the control input $u_i(k)$ by (6).
(2) The MAS state $x(k)$ is evolved to $x(k+1)$ by (2).
(3) Measure the state of neighbouring agents, $Z_i(k+1)$ by (3).
(4) Update ${}^i\hat{x}(k+1)$ using (8), and (5).

$k \leftarrow k + 1$

end while

Remark 5. The taxonomy of centralized and distributed methodologies for MAS can be distinctly brought into play across design and operational phases. In the design phase, distributed methods rely solely on information from neighbouring agents, allowing each agent to compute solutions independently in a distributed manner. While this method provides robustness and scalability, it is difficult to assure performance without access to complete system information. Conversely, our centralized design is favoured for its ability to leverage full system information during the offline design phase, thereby optimizing complex MAS missions [36–39, 47]. For the operational phase, our method adopts distributed control that depends only on local measurements from neighbouring agents, unlike centralized control, which demands full MAS measurements and becomes impractical for MAS with network topological constraints.

3.4 | Theoretic stability analysis

To analyse the system stability in the MAS context, we refer to the notion of semistability. Compared with the conventional stability concept, semistability implies closed-loop dynamics reaching a continuum of equilibria depicting designed states of convergence. In particular, it has the property that the resulting response of the system is determined not only by the system dynamics but also by initial MAS states, which is more relevant to describing the behaviour of MAS [69]. The formal definition of semistability is as follows.

Definition 2 [69]. The system $x(k+1) = Ax(k)$ is semistable if for every $\lambda \in spec(A)$, the condition $|\lambda| < 1$ or $\lambda = 1$ is semisimple.

To this end, we resort to some of the semistability results as follows.

Definition 3 [70]. The pair $(\tilde{A}, \tilde{B})$ is semicontrollable if

$$\left(\bigcap_{i=1}^{n} \mathcal{N}(\tilde{B}^{\mathrm{T}}(\tilde{A}^{\mathrm{T}} - I_n)^{i-1})\right)^{\perp} = \left(\mathcal{N}(\tilde{A}^{\mathrm{T}} - I_n)\right)^{\perp}$$

where $(\tilde{A}^{\mathrm{T}} - I_n)^0 = I_n$.

Definition 4 [70]. Let $\tilde{C} \in \mathbb{R}^{Nn \times Nn}$. The pair $(\tilde{A}, \tilde{C})$ is semiobservable if

$$\left(\bigcap_{i=1}^{n} \mathcal{N}(\tilde{C}(\tilde{A} - I_n)^{i-1})\right) = \mathcal{N}(\tilde{A} - I_n).$$

Here, semicontrollability and semiobservability describe the system's ability to deal with the equilibrium manifold instead of equilibrium points.

Accordingly, the stability analysis of the proposed distributed control law F^* is condensed into checking the first moment of (7), expressed by

$$\mathbb{E}[x(k+1)] = (\tilde{A} + \tilde{B}F^*)\mathbb{E}[x(k)], \tag{28}$$

is semistable, that is, $(\tilde{A} + \tilde{B}F^*)$ is semistable.

Theorem 1 [70]. *Consider the closed-loop system* G *as follows:*

$$x(k+1) = (\tilde{A} + \tilde{B}F^*)x(k).$$

Then G *is semistable if and only if for the semicontrollable pair* $(\tilde{A}, \tilde{B})$, *and semiobservable pair* $(\tilde{A}, \tilde{C})$ *there exists a* $P \geq 0$ *such that*

$$P = (\tilde{A} + \tilde{B}F^*)^{\mathrm{T}} P(\tilde{A} + \tilde{B}F^*) + Q + F^{*\mathrm{T}} R F^* \tag{29}$$

where $R > 0$, *and* $Q = \tilde{C}^{\mathrm{T}}\tilde{C} \geq 0$.

We are now ready to present the semistability result for our designed control gain.

Theorem 2. *Given the stochastic MAS dynamics (7), whose first moment equation is expressed by (28), the steady-state distributed control law* F^* *is semistable if the matrix pair* $(\tilde{A}, \tilde{B})$ *is semicontrollable, and* $(\tilde{A}, Q^{\frac{1}{2}})$ *is semiobservable.*

Proof. Since the designed $P^{(\bar{k})}$ and F^* satisfies (29), the proof is a direct consequence of Theorem 1. □

4 | NUMERICAL SIMULATION

The proposed distributed control-estimation synthesis is validated using a numerical simulation of a multi-vehicle formation manoeuvre. The formation manoeuvre aims to arrange N homogeneous vehicles into a rigid circular shape, ensuring equal spacing between neighbouring vehicles, as shown by the formation geometry in Figure 2. To quantify this formation goal, we define a set of relative positions between vehicles, denoted as $x_{ij}^{\mathrm{ref}}, \forall i, j \in \mathcal{V}$, representing the desired relative position between the vehicle j and the vehicle i. Subsequently, the global quadratic cost for the formation manoeuvre of the MAS is formulated as,

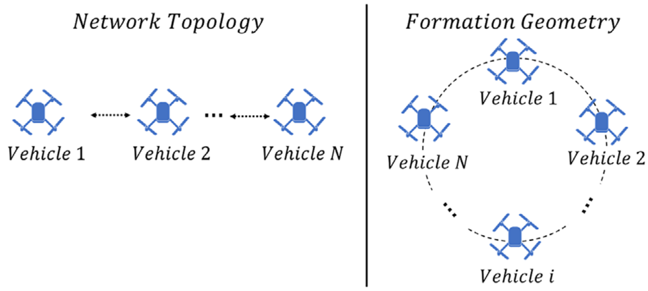

FIGURE 2 Multi-vehicle circular formation manoeuvre under undirected string network topology.

$$\mathcal{J} := \lim_{T\to\infty} \frac{1}{T} \sum_{k=0}^{T} \mathbb{E}\left[\left(\tilde{x}(k)^{\mathrm{T}} Q \tilde{x}(k) + u(k)^{\mathrm{T}} R u(k)\right)\right], \tag{30}$$

$$\tilde{x}(k) = \left[\tilde{x}_1^{\mathrm{T}}(k) \quad \tilde{x}_2^{\mathrm{T}}(k) \quad \cdots \quad \tilde{x}_N^{\mathrm{T}}(k)\right]^{\mathrm{T}},$$

$$\tilde{x}_i(k) = x_i(k) - {}^{i}x^{\mathrm{ref}}, \forall i \in \mathcal{V},$$

$${}^{i}x^{\mathrm{ref}} = \left[x_{i1}^{\mathrm{refT}}\ x_{i2}^{\mathrm{refT}}\ \cdots\ x_{iN}^{\mathrm{refT}}\right]^{\mathrm{T}},$$

where $Q = \mathcal{L} \otimes I_4$, $R = I_N \otimes I_2$. And considering two-dimensional position and velocity as the state of each vehicle, the dynamics of the individual vehicle is governed by (1), where,

$$A = \begin{bmatrix} 1 & T_s & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & T_s \\ 0 & 0 & 0 & 1 \end{bmatrix}, \ B = \begin{bmatrix} \frac{T_s^2}{2} & 0 \\ T_s & 0 \\ 0 & \frac{T_s^2}{2} \\ 0 & T_s \end{bmatrix}.$$

and the sampling time, denoted as T_s, is set to 0.1 s. Disturbances and measurement noises, in square meters m^2, are uniformly set as $\Theta_i = 0.1 \times I_4$ and $\Xi_{ij} = 0.1 \times I_4, \forall i, j \in \mathcal{V}$.

As shown in Figure 2, the network topology is given as an undirected string graph $(1 \leftrightarrow 2 \leftrightarrow \cdots \leftrightarrow N)$, representing a linear sequence of N vertices connected in series. As to the theoretical analysis, the matrix pairs $(\tilde{A}, \tilde{B})$ and $(\tilde{A}, Q^{\frac{1}{2}})$ are semicontrollable and semiobservable by Definitions 3 and 4, respectively. Then by Theorem 2, we can guarantee that the first moment of MAS is semistable, which ensures the stable formation manoeuvre of vehicles.

We begin our analysis with the proposed distributed control law in a system consisting of five vehicles ($N = 5$). The distributed control gain matrix governs interactions among vehicles and is segmented into smaller block matrices, each

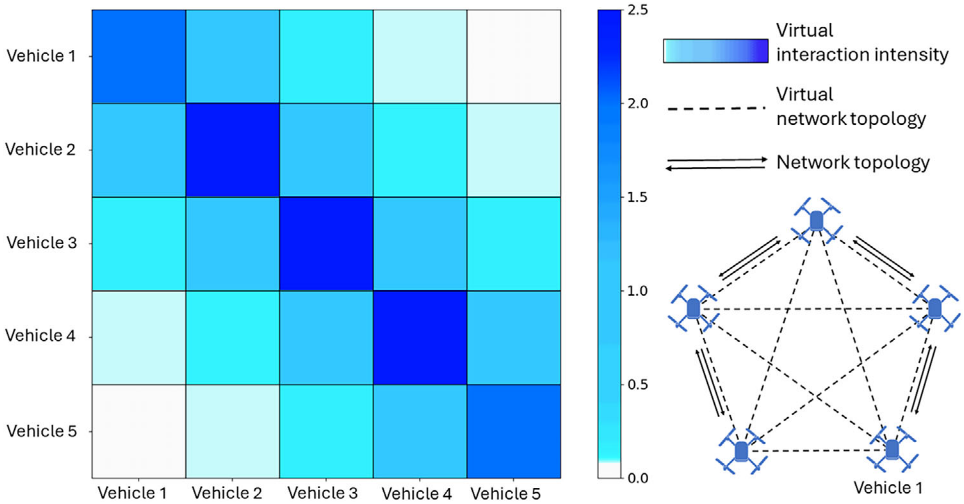

FIGURE 3 Visualization of the virtual interaction intensities among five vehicles forming a circular shape via the proposed distributed control, operating under uniform measurement noise condition.

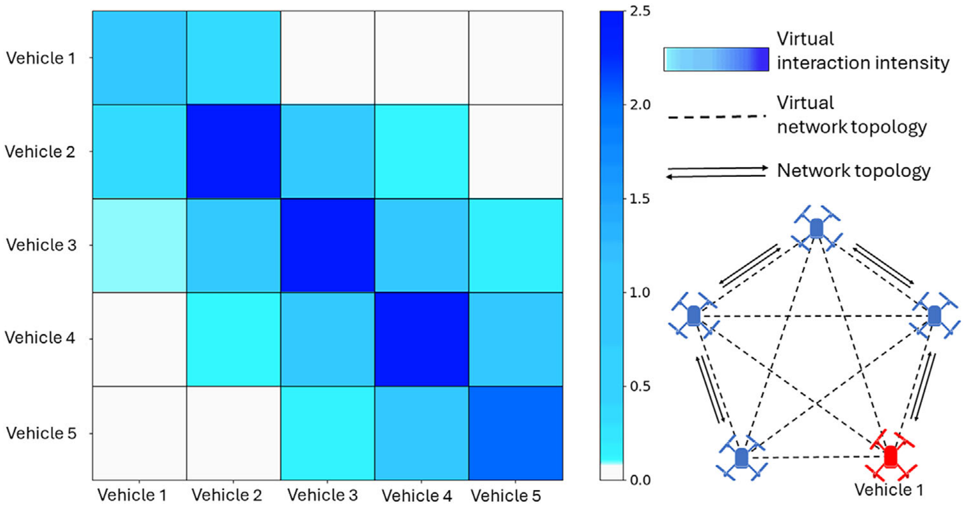

FIGURE 4 Visualization of the virtual interaction intensities when the measurement noises of vehicle 1 are tenfold higher than the others.

signifying the interaction between a specific pair of vehicles. Computing the norm of these block matrices allows us to quantify the interaction intensity between each pair of vehicles, as illustrated in Figure 3. For instance, the top row illustrates the virtual interaction intensities between vehicle 1 and others from vehicle 1's perspective. Notably, substantially large interaction intensities correspond to non-zero entries in the Laplacian matrix $\mathcal{L}$, indicating intensive interactions among neighbouring vehicles. Even in cases of zero entries in $\mathcal{L}$, the intensity values remain non-zero, signifying virtual interactions among non-neighbouring vehicles enacted by the fully connected virtual network topology established through the distributed estimator.

To further elucidate the fundamental mechanism behind the proposed distributed control-estimation synthesis, additional simulations are conducted with the different measurement noise configurations. Specifically, the noise covariance for measurements of vehicle 1 is scaled to be ten times greater than that of the other vehicles, such that $\Xi_{i1} = I_4, \forall i \in \mathcal{V}$. Figure 4 displays the resulting virtual interaction intensities. Notably, augmenting the noise affecting vehicle 1 leads to a reduction in interaction intensities between vehicle 1 and all other vehicles, that is, the first row and the first column of the grid in Figure 4. This arises from the fact that the optimization of the control gain considers estimation errors from all individual vehicles within the cost function. Consequently, the control gain is designed to mitigate the impact of estimation errors due to measurement noise. This trend leads to the intuition that reduced interaction is favoured when the measured information is less accurate.

For the purpose of comparative performance analysis, we evaluate our method against three distinct approaches. The first comparison method is a centralized output feedback designed by linear quadratic Gaussian (LQG) control framework, detailed in [59]. This utilizes all available measurements within the network to update centralized estimates, and each vehicle shares

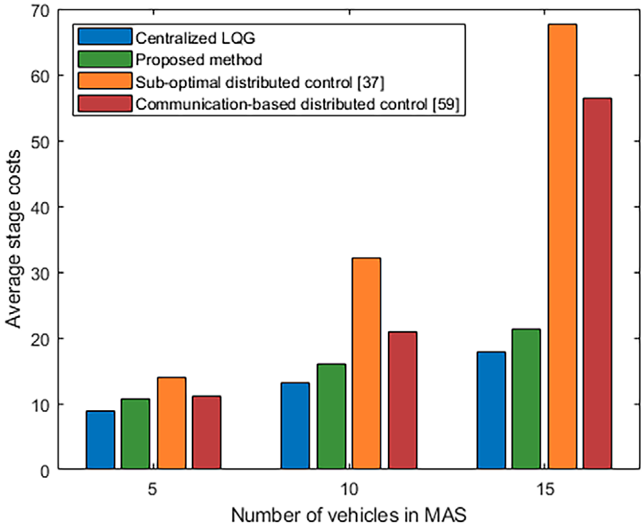

FIGURE 5 Average stage costs of formation manoeuvre with varying number of vehicles (Monte Carlo simulations with 1000 runs). LQG, linear quadratic Gaussian; MAS, multi-agent system.

this information for centralized optimal control. Although it is not a distributed control, this method can be a good benchmark to compare with the distributed control schemes. The second comparison method is a suboptimal distributed control method introduced in [37], where only neighbouring vehicles' measurements are directly utilized to compute distributed control inputs. The third comparison method employs a distributed estimation-based control approach, described in [59]. This method introduces communication between neighbouring vehicles to exchange MAS estimates. Through γ intermediate communications per sampling time, vehicles receive distributed estimates from their neighbours, perform consensus between their estimates, and subsequently engage in distributed control based on their own estimates. As the size of the network gets bigger, that is, with a bigger N, more intermediate communications are required to reach a reasonable degree of consensus. These three comparison methods, along with our proposed method, are evaluated via Monte Carlo simulations for $N = 5, 10$ and 15. Throughout the simulations, the network topology is kept as an undirected string graph while the formation goal is circular geometry. To secure the consensus on the estimates for stable operation with the third comparison method across varying vehicle numbers, we adjust γ to 5, 20 and 55 accordingly.

Figure 5 shows the average stage costs over 1000 Monte Carlo runs, demonstrating our distributed control-estimation law's effectiveness, which nearly achieves centralized optimal control performance (the first comparison method) and surpasses the second and the third comparison methods. In particular, the average costs of the both second and third methods significantly increase when the number of agents exceeds 10, signifying poor efficiency in larger networks. Furthermore, the additional intermediate communication overhead incurred by the third method makes it more challenging to deploy in the real world. Meanwhile, our method scales well, closely matching the performance of the centralized optimal solution's performance without requiring additional communication, demonstrating its effectiveness across various network sizes and its applicability to MAS with larger networks.

Table 1 presents a comparison of the scalability between the comparison methods and ours in relation to the number of agents and states, remarking on evaluation criteria such as the computational complexities during the design and operational phases, the load associated with acquiring measurements from neighbours, and the load from additional intermediate communications per sampling time. These criteria are assessed using big $\mathcal{O}$ notation which is desired to be reduced in practical applications. In terms of computational complexity, the second comparison method stands out, exhibiting the minimum load and computational complexity during both the design and operational phases, albeit at poor performance. In terms of measurement load, the centralized LQG ranks least favourably, as it necessitates processing measurements from the entire MAS. Our proposed method and the second comparison method offer significant advantages in terms of communication efficiency, as neither requires additional intermediate communication. This contrasts with the third comparison method, whose intermediate communication load scales poorly. In summary, our method closely approximates the centralized optimal solution, simultaneously eliminating communication overhead and minimizing risks associated with communication vulnerabilities by utilizing only neighbouring measurements.

5 | HARDWARE EXPERIMENT

To further validate the practical soundness of the proposed method, experiments involving the formation flight of five quadrotors in an indoor test-bed are conducted, as portrayed in Figure 7. For these experiments, five Crazyflie quadrotors are controlled remotely through the Crazyswarm platform [71]. The source code and experiment video can be found in the git repository. https://github.com/HMCL-UNIST/MAS-Distributed-Ctrl-Est-Synthesis.git

Considering the velocity for controlling the quadrotors, we model MAS dynamics using velocity in the horizontal plane as control input. Each quadrotor's state is defined by its horizontal position, $x = [p_x^{\mathrm{T}}\ p_y^{\mathrm{T}}] \in \mathbb{R}^2$ (meters), leading to MAS dynamics represented by $A = I_2$ and $B = I_2 \times T_s$, with $T_s =$ 0.05 s. The rest of the set-up is akin to the numerical simulations, that is, undirected string network topology and the circular formation geometry according to the cost (30) with $Q = \mathcal{L} \otimes I_2,\ \ R = I_5 \otimes I_2$. Measurement data acquisition for each quadrotor is performed using an Optitrak motion capture system operating at a 100 Hz frequency. Note that the position measurement obtained by Optitrack exhibits mm-level accuracy, which can be used as ground truth when evaluating the performance. To emulate sensor noise for our experiment, we infuse synthetic zero-mean Gaussian noise with 0.1 m^2 variance ($\Xi_{ij} = 0.1 \times I_2, \forall i, j \in \mathcal{V}$) into the Optitrack measurements before feeding them into the distributed estimator. As to the process noise and/or disturbance of the quadrotor, we

TABLE 1 Comparative analysis of computational complexity, measurement, and additional communication loads

Algorithms	Centralized LQG	Proposed method	Suboptimal distributed control [37]	Communication-based distributed control [59]
Computation complexity (design phase)	$\mathcal{O}(n^3N^3)$	$\mathcal{O}(n^3N^3)$	$\mathcal{O}(n^3)$	$\mathcal{O}\left(n^3\left(\sum_{i,j}^{N} a_{ij}\right)^3\right)$
Computation complexity per agent (operational phase)	$\mathcal{O}(n^2N^2)$	$\mathcal{O}(n^2N^2)$	$\mathcal{O}(n^2\lvert\Omega_i\rvert^2)$	$\mathcal{O}(n^2N^2)$
Load of measurements per sampling time	$\mathcal{O}(N^2)$	$\mathcal{O}\left(\sum_{i,j}^{N} a_{ij}\right)$	$\mathcal{O}\left(\sum_{i,j}^{N} a_{ij}\right)$	$\mathcal{O}\left(\sum_{i,j}^{N} a_{ij}\right)$
Load of additional communication per sampling time	—	—	—	$\mathcal{O}\left(\gamma\sum_{i,j}^{N} a_{ij}\right)$

Abbreviation: LQG, linear quadratic Gaussian.

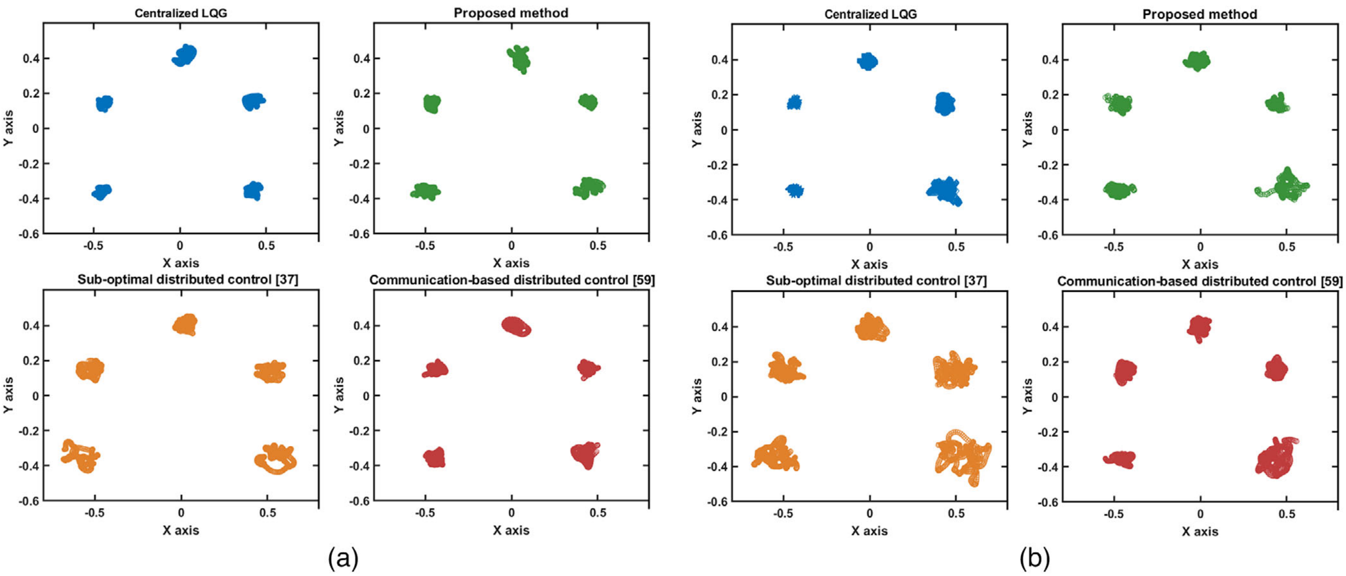

FIGURE 6 Traces of positions relative to the center of multi-agent system (MAS) from real-world formation flight experiments, with quadrotor 1's position measurement noise characterized by variances of (a) 0.2 m^2 and (b) 0.5 m^2. Blue: Centralized linear quadratic Gaussian (LQG); Green: Proposed method; Orange: Suboptimal distributed control [37]; Red: Communication-based distributed control [59].

test its hovering manoeuvre in our indoor test-bed and found that the positioning statistics exhibit 0.05 m^2 variance. This is used as the disturbance covariance for the distributed estimator ($\Theta_i = 0.05 \times I_2, \forall i \in \mathcal{V}$).

To assess the effectiveness of our method amidst varying measurement noise levels, experiments were conducted by altering the noise level of quadrotor 1 in two configurations: one with a variance of 0.2 m^2, that is, ($\Xi_{i1} = 0.2 \times I_2, \forall i \in \mathcal{V}$), and 0.5 m^2, that is, ($\Xi_{i1} = 0.5 \times I_2, \forall i \in \mathcal{V}$). Given the control-estimation laws designed by Algorithm 1, control commands are computed and transmitted to each quadrotor at a frequency of 20 Hz, using a laptop powered by an Intel Core i7-122 CPU.

Comparative analysis is conducted with the comparison methods detailed in Section 4, conducting ten trials per method, with each trial lasting 10 s of flight. This duration is enough to observe the quality of formation behaviour toward the target geometry. For the third comparison method, the number of intermediate communication steps between neighbouring quadrotors is set as $\gamma = 3$. Figure 6a,b shows the quadrotors'

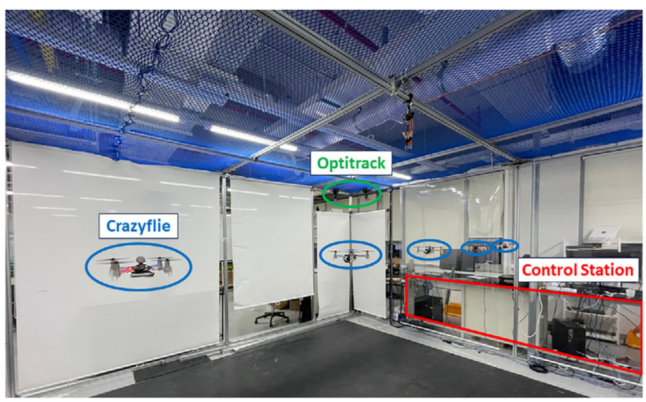

FIGURE 7 Indoor testing environment (6 m × 6 m horizontal space) for rigid shape formation flight with five Crazyflie quadrotors and a motion tracking system. To ensure formation, quadrotors are set to maintain an equal distance between neighbouring while aligning in a circular pattern (radius of 0.5 m).

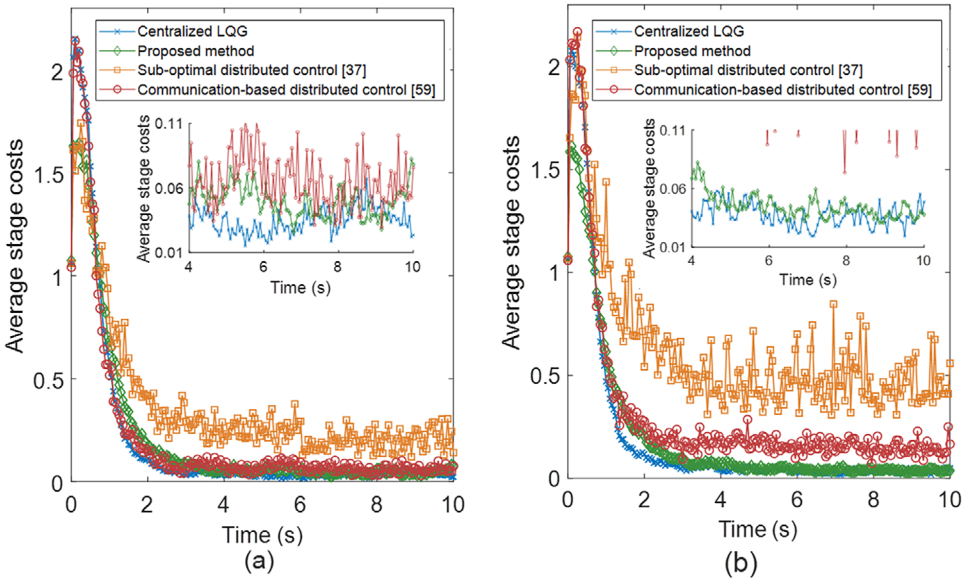

FIGURE 8 Average stage costs from formation manoeuvres (ten trials each lasting 10 s), with quadrotor 1's position measurement noise with a variance of (a) 0.2 m^2 and (b) 0.5 m^2. LQG, linear quadratic Gaussian.

position traces relative to the center of the MAS, highlighting how the formation aligns with the objective shape. The second comparison method's traces are notably more dispersed than others, and the third comparison method's dispersion increases with higher noise levels. On the other hand, our proposed method performs a decent formation almost matched to the centralized optimal solution, underlining its robustness against various noise levels. Figure 8a,b, respectively depicts the average stage costs of the methods over time, under two different noise configurations for quadrotor 1. The second comparison method performs poorly, while the third comparison method initially matches our performance under low noise but degrades with increased noise. Remarkably, our method matches the performance of the centralized optimal solution in both scenarios without relying on additional intermediate communications, demonstrating its efficiency and robustness against noise variations.

6 | CONCLUSION

In this study, we investigated the optimal distributed control-estimation synthesis problem for stochastic linear MAS, addressing complexities imposed by network topological constraints. An iterative optimization process is proposed to jointly design distributed linear control and estimation laws while considering their mutual influence. Furthermore, we verify the stability of our proposed solutions through rigorous theoretical proof. The designed distributed control and estimation laws were validated through numerical simulations and real-hardware quadrotor formation flight experiments. The results consistently showed that the proposed distributed estimation-based control outperformed existing methods in terms of performance without requiring additional communication load. This approach ensures the prevention of communication congestion as the network size scales. This work paves the way for a new research paradigm in optimal distributed control, with many future directions including:

(1) Extending the proposed distributed control-estimation framework to address real-world challenges such as time-varying network topology, physical constraints, and process delays;
(2) Developing robust and plug-and-play control-estimation synthesis for cases when the given dynamics model or the network topology is inaccurate and/or partially known;
(3) Advancing scalability for larger networked systems by reducing the computational overhead of each agent; and
(4) Applying the proposed distributed control-estimation framework to facilitate the cooperative MAS mission with complex dynamics, for example, coordinated load transfer using multiple aerial vehicles.

AUTHOR CONTRIBUTIONS

Hojin Lee: Conceptualization; data curation; formal analysis; investigation; methodology; resources; software; validation; visualization; writing—original & draft. **Chanyong Lee**: Data curation; software; validation; visualization. **Jusang Lee**: Data curation; software; validation; visualization. **Cheolhyeon Kwon**: Funding acquisition; investigation; project administration; supervision; writing—review and editing.

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CONFLICT OF INTEREST STATEMENT

The authors declare no conflicts of interests.

DATA AVAILABILITY STATEMENT

The data that support the findings of this study are available from the corresponding author upon reasonable request.

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